

# A NEW GENERALIZED ALGEBRA FOR THE BALANCING OF $\aleph$ CHEMICAL REACTIONS

## NOVA POSPLOŠENA ALGEBRA ZA URAVNOTEŽENJE KEMIJSKIH REAKCIJ $\aleph$

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Prejem rokopisa – received: 2012-12-01; sprejem za objavo – accepted for publication: 2013-07-15

In this article we develop a new generalized algebra for the balancing of  $\aleph$  chemical reactions. This is a completely new approach to the balancing of these kinds of chemical reactions that is based on an understanding of reaction analysis and the elementary theory of inequalities. The generators of the reaction determined all the interactions among the stoichiometric coefficients.

Keywords:  $\aleph$  chemical reactions, generalized algebra, balancing reactions

V tem članku smo razvili novo posplošeno algebro za uravnoteženje kemijskih reakcij. To je popolnoma nov način uravnoteženja kemijskih reakcij, ki temelji na navidezni analizi reakcij in elementarni teoriji neenakosti. Generatorji reakcij določajo vse interakcije med stehiometričnimi koeficienti.

Ključne besede: kemijske reakcije  $\aleph$ , posplošena algebra, uravnoteženje reakcij

## 1 INTRODUCTION

Since the balancing of chemical reactions in chemistry is a basic and fundamental issue it deserves to be considered on a satisfactory level. This topic always draws the attention of students and teachers, but it is never a finished product. Because of its importance in chemistry and mathematics, there are several articles devoted to the subject. However, here we will not provide a historical perspective about this topic, because it has been done in so many previous publications. We can, however, still provide a full balancing of chemical reactions with the use of a generalized algebra.

In mathematics and chemistry there are several mathematical methods for balancing chemical reactions.<sup>1-7</sup> All of them are based on generalized matrix inverses and they have formal scientific properties that need a higher level of mathematical knowledge for their application. The so-called *chemical methods* are paradoxical and out of order.<sup>8</sup>

The newest approach for balancing  $\aleph$  reactions is developed in<sup>9</sup>. The present article is a prolongation of the previous research.<sup>9,10</sup>

Generally speaking, balancing a chemical reaction that possesses atoms with fractional oxidation numbers is a tough problem in chemistry. It is really hard for reactions that have only one set of coefficients, but for  $\aleph$  reactions that have an infinite number of sets of coefficients, this problem is extremely hard.<sup>11,12</sup>

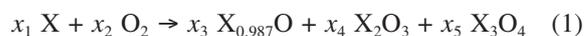
In the next section we shall consider three general reactions of oxidation. They are examples of *elementary*  $\aleph$  reactions, which possess atoms with fractional and

integer oxidation numbers. Actually, we balanced three general  $\aleph$  reactions with one, two and three arbitrary elements. The first reaction plays a very important role in metallurgy. For instance, this reaction is a basic reaction in the theory of metal corrosion, ferrous metallurgy as well as the theory of metallurgical processes, but unfortunately it was not taken into account until today. The main reason why this reaction was neglected lies in its balancing. This article will provide its full balancing, which is neither easy nor simple.

## 2 MAIN RESULTS

Now we shall consider the announced  $\aleph$  reactions.

**Reaction 1.** Let us consider this general  $\aleph$  reaction with one arbitrary element:



The above chemical reaction (1) reduces to the following system of linear equations:

$$\begin{aligned} x_1 &= 0.987x_3 + 2x_4 + 3x_5, \\ 2x_2 &= x_3 + 3x_4 + 4x_5 \end{aligned} \quad (2)$$

Since the system (2) has two linear equations and five unknowns, we can solve it in  $5!/[(5-2)!] = 10$  ways. Actually, we shall determine all the possible general solutions of the system (2). They are the following pairs:  $(x_1, x_2)$ ,  $(x_1, x_3)$ ,  $(x_1, x_4)$ ,  $(x_1, x_5)$ ,  $(x_2, x_3)$ ,  $(x_2, x_4)$ ,  $(x_2, x_5)$ ,  $(x_3, x_4)$ ,  $(x_3, x_5)$  and  $(x_4, x_5)$ .

<sup>1°</sup> Let  $x_3$ ,  $x_4$  and  $x_5$  be arbitrary real numbers, then the general solution of the system (2) is:

$$\begin{aligned}x_1 &= 0.987x_3 + 2x_4 + 3x_5, \\x_2 &= x_3/2 + 3x_4/2 + 2x_5\end{aligned}\quad (3)$$

After the substitution of (3) into (1), the balanced reaction takes on this general form:

$$\begin{aligned}(0.987x_3 + 2x_4 + 3x_5) \mathbf{X} + (x_3/2 + 3x_4/2 + 2x_5) \mathbf{O}_2 \\ \rightarrow x_3 \mathbf{X}_{0.987}\mathbf{O} + x_4 \mathbf{X}_2\mathbf{O}_3 + x_5 \mathbf{X}_3\mathbf{O}_4, \\ \forall x_3, x_4, x_5 \in \mathfrak{R}.\end{aligned}\quad (4)$$

This means that by finding the coefficients of the products we find the coefficients of the reactants.

2° Assume  $x_2$ ,  $x_4$  and  $x_5$  are arbitrary real numbers, then the general solution of the system (2) is:

$$\begin{aligned}x_1 &= 1.977x_2 - 0.961x_4 - 0.948x_5, \\x_3 &= 2x_2 - 3x_4 - 4x_5\end{aligned}\quad (5)$$

The balanced reaction (1) obtains this general form:

$$\begin{aligned}(1.977x_2 - 0.961x_4 - 0.948x_5) \mathbf{X} + x_2 \mathbf{O}_2 \\ \rightarrow (2x_2 - 3x_4 - 4x_5) \mathbf{X}_{0.987}\mathbf{O} + x_4 \mathbf{X}_2\mathbf{O}_3 + x_5 \mathbf{X}_3\mathbf{O}_4\end{aligned}\quad (6)$$

Since the generators (5) are positive, it should immediately follow these inequalities:

$$\begin{aligned}1.977x_2 - 0.961x_4 - 0.948x_5 > 0, \\2x_2 - 3x_4 - 4x_5 > 0\end{aligned}\quad (7)$$

From (7) we obtain the inequality:

$$x_2 > 1.5x_4 + 2x_5\quad (8)$$

The expression (8) is a necessary and sufficient condition for a general reaction (6) to hold. In other words, the reaction is possible if and only if the condition (8) is satisfied.

3° Suppose  $x_2$ ,  $x_3$  and  $x_5$  are arbitrary real numbers. The general solution of the system (2) is:

$$\begin{aligned}x_1 &= 4x_2/3 + 0.961x_3/3 + x_5/3, \\x_4 &= 2x_2/3 - x_3/3 - 4x_5/3\end{aligned}\quad (9)$$

If we substitute (9) into (1), the general form of the balanced reaction is:

$$\begin{aligned}(4x_2/3 + 0.961x_3/3 + x_5/3) \mathbf{X} + x_2 \mathbf{O}_2 \\ \rightarrow x_3 \mathbf{X}_{0.987}\mathbf{O} + (2x_2/3 - x_3/3 - 4x_5/3) \mathbf{X}_2\mathbf{O}_3 + x_5 \mathbf{X}_3\mathbf{O}_4\end{aligned}\quad (10)$$

Since the generator  $x_4 > 0$ , it should immediately follow that:

$$x_2 > 0.5x_3 + 2x_5\quad (11)$$

The reaction (10) is possible if and only if the condition (11) is satisfied. The inequality (11) is a necessary and sufficient condition to hold (10).

4° Let  $x_2$ ,  $x_3$  and  $x_4$  be arbitrary real numbers. The general solution of the system (2) is:

$$\begin{aligned}x_1 &= 3x_2/2 + 0.948x_3/4 - x_4/4, \\x_5 &= x_2/2 - x_3/4 - 3x_4/4\end{aligned}\quad (12)$$

After the substitution of (12) into (1), the general chemical reaction takes on this form:

$$\begin{aligned}(3x_2/2 + 0.948x_3/4 - x_4/4) \mathbf{X} + x_2 \mathbf{O}_2 \\ \rightarrow x_3 \mathbf{X}_{0.987}\mathbf{O} + x_4 \mathbf{X}_2\mathbf{O}_3 \\ + (x_2/2 - x_3/4 - 3x_4/4) \mathbf{X}_3\mathbf{O}_4\end{aligned}\quad (13)$$

Since the generators  $x_1$ ,  $x_5 > 0$ , then it must be:

$$\begin{aligned}3x_2/2 + 0.948x_3/4 - x_4/4 > 0, \\x_2/2 - x_3/4 - 3x_4/4 > 0\end{aligned}\quad (14)$$

After (14) it immediately follows that:

$$x_2 > x_3/2 + 3x_4/2\quad (15)$$

The inequality (15) is a necessary and sufficient condition to hold the general reaction (13), *i.e.*, the reaction (13) holds if and only if (15) is satisfied.

5° Suppose  $x_1$ ,  $x_4$  and  $x_5$  are arbitrary real numbers. The general solution of the system (2) is:

$$\begin{aligned}x_2 &= x_1/1.974 + 0.961x_4/1.974 + 0.948x_5/1.974, \\x_3 &= x_1/0.987 - 2x_4/0.987 - 3x_5/0.987\end{aligned}\quad (16)$$

The balanced chemical reaction (1) obtains this general form:

$$\begin{aligned}x_1 \mathbf{X} + (x_1/1.974 + 0.961x_4/1.974 + 0.948x_5/1.974) \mathbf{O}_2 \\ \rightarrow (x_1/0.987 - 2x_4/0.987 - 3x_5/0.987) \mathbf{X}_{0.987}\mathbf{O} \\ + x_4 \mathbf{X}_2\mathbf{O}_3 + x_5 \mathbf{X}_3\mathbf{O}_4\end{aligned}\quad (17)$$

Since the generator  $x_3 > 0$ , then it must be:

$$x_1 > 2x_4 + 3x_5\quad (18)$$

The above inequality (18) is a necessary and sufficient condition to hold (17), *i.e.*, the reaction (17) holds if and only if (18) is satisfied.

6° Assume  $x_1$ ,  $x_3$  and  $x_5$  are arbitrary real numbers. The general solution of the system (2) is:

$$\begin{aligned}x_2 &= 3x_1/4 - 0.961x_3/4 - x_5/4, \\x_4 &= x_1/2 - 0.987x_3/2 - 3x_5/2\end{aligned}\quad (19)$$

The balanced chemical reaction (1) has this general form:

$$\begin{aligned}x_1 \mathbf{X} + (3x_1/4 - 0.961x_3/4 - x_5/4) \mathbf{O}_2 \\ \rightarrow x_3 \mathbf{X}_{0.987}\mathbf{O} + (x_1/2 - 0.987x_3/2 \\ - 3x_5/2) \mathbf{X}_2\mathbf{O}_3 + x_5 \mathbf{X}_3\mathbf{O}_4\end{aligned}\quad (20)$$

Since the generators  $x_2$ ,  $x_4 > 0$ , then it must be that:

$$\begin{aligned}3x_1 - 0.961x_3 - x_5 > 0, \\x_1 - 0.987x_3 - 3x_5 > 0\end{aligned}\quad (21)$$

From (21) we obtain:

$$x_1 > 0.987x_3 + 3x_5\quad (22)$$

The above expression (22) is a necessary and sufficient condition to hold the general reaction (20), *i.e.*, the reaction (20) holds if and only if (22) is satisfied.

7° Assume  $x_1$ ,  $x_3$  and  $x_4$  are arbitrary real numbers. The general solution of the system (2) is:

$$\begin{aligned}x_2 &= 2x_1/3 - 0.474x_3/3 + x_4/6, \\x_5 &= x_1/3 - 0.987x_3/3 - 2x_4/3\end{aligned}\quad (23)$$

The balanced chemical reaction (1) has this general form:

$$\begin{aligned}x_1 \mathbf{X} + (2x_1/3 - 0.474x_3/3 + x_4/6) \mathbf{O}_2 \\ \rightarrow x_3 \mathbf{X}_{0.987}\mathbf{O} + x_4 \mathbf{X}_2\mathbf{O}_3 \\ + (x_1/3 - 0.987x_3/3 - 2x_4/3) \mathbf{X}_3\mathbf{O}_4\end{aligned}\quad (24)$$

Since the generators  $x_2$ ,  $x_5 > 0$ , then these inequalities should follow:

$$\begin{aligned} 4x_1 - 0.948x_3 + x_4 &> 0, \\ x_1 - 0.987x_3 - 2x_4 &> 0 \end{aligned} \quad (25)$$

From (25) we obtain:

$$x_1 > 0.987x_3 + 2x_4 \quad (26)$$

The inequality (26) is a necessary and sufficient condition to hold the general reaction (24), *i.e.*, the reaction (24) holds if and only if (26) is satisfied.

8° Let us assume  $x_1$ ,  $x_2$  and  $x_5$  are arbitrary real numbers. The general solution of the system (2) is:

$$\begin{aligned} x_3 &= 3x_1/0.961 - 4x_2/0.961 - x_5/0.961, \\ x_4 &= -x_1/0.961 + 1.974x_2/0.961 \\ &\quad - 0.948x_5/0.961 \end{aligned} \quad (27)$$

After the substitution of (27) into (1), the general chemical reaction obtains this form:

$$\begin{aligned} x_1 \mathbf{X} + x_2 \mathbf{O}_2 &\rightarrow (3x_1/0.961 - 4x_2/0.961 \\ &- x_5/0.961) \mathbf{X}_{0.987}\mathbf{O} + (-x_1/0.961 + 1.974x_2/0.961 \\ &- 0.948x_5/0.961) \mathbf{X}_2\mathbf{O}_3 + x_5 \mathbf{X}_3\mathbf{O}_4 \end{aligned} \quad (28)$$

Since the generators  $x_3$ ,  $x_4 > 0$ , then these inequalities should follow:

$$\begin{aligned} 3x_1 - 4x_2 - x_5 &> 0, \\ -x_1 + 1.974x_2 - 0.948x_5 &> 0 \end{aligned} \quad (29)$$

From (29) we obtain:

$$\begin{aligned} 4x_2/3 + x_5/3 < x_1 < 1.974x_2 - 0.948x_5, \\ x_2 > 2x_5 \end{aligned} \quad (30)$$

The inequalities (30) are necessary and sufficient conditions to hold the general reaction (28). In other words, the reaction (28) holds if and only if (30) are satisfied.

9° Suppose  $x_1$ ,  $x_2$  and  $x_4$  are arbitrary real numbers. The general solution of the system (2) is:

$$\begin{aligned} x_3 &= 4x_1/0.948 - 6x_2/0.948 + x_4/0.948, \\ x_5 &= -x_1/0.948 + 1.974x_2/0.948 - 0.961x_4/0.948 \end{aligned} \quad (31)$$

The balanced chemical reaction (1) has this general form:

$$\begin{aligned} x_1 \mathbf{X} + x_2 \mathbf{O}_2 &\rightarrow (4x_1/0.948 - 6x_2/0.948 \\ &- x_4/0.948) \mathbf{X}_{0.987}\mathbf{O} + x_4 \mathbf{X}_2\mathbf{O}_3 + (-x_1/0.948 \\ &+ 1.974x_2/0.948 - 0.961x_4/0.948) \mathbf{X}_3\mathbf{O}_4 \end{aligned} \quad (32)$$

Since the generators  $x_3$ ,  $x_5 > 0$ , then these inequalities should follow:

$$\begin{aligned} 4x_1 - 6x_2 - x_4 &> 0, \\ -x_1 + 1.974x_2 - 0.961x_4 &> 0 \end{aligned} \quad (33)$$

From (33) we obtain:

$$\begin{aligned} 3x_2/2 - x_4/4 < x_1 < 1.974x_2 - 0.961x_4, \\ x_2 > 3x_4/2 \end{aligned} \quad (34)$$

The inequalities (34) are necessary and sufficient conditions to hold the general reaction (32), *i.e.*, the reaction (32) holds if and only if (34) are satisfied.

10° Let us assume  $x_1$ ,  $x_2$  and  $x_3$  are arbitrary real numbers. The general solution of the system (2) is:

$$\begin{aligned} x_4 &= -4x_1 + 6x_2 + 0.948x_3, \\ x_5 &= 3x_1 - 4x_2 - 0.961x_3 \end{aligned} \quad (35)$$

The balanced chemical reaction (1) has this general form:

$$\begin{aligned} x_1 \mathbf{X} + x_2 \mathbf{O}_2 &\rightarrow x_3 \mathbf{X}_{0.987}\mathbf{O} \\ &+ (-4x_1 + 6x_2 + 0.948x_3) \mathbf{X}_2\mathbf{O}_3 \\ &+ (3x_1 - 4x_2 - 0.961x_3) \mathbf{X}_3\mathbf{O}_4 \end{aligned} \quad (36)$$

Since the generators  $x_4$ ,  $x_5 > 0$ , then follow these inequalities:

$$\begin{aligned} -4x_1 + 6x_2 + 0.948x_3 &> 0, \\ 3x_1 - 4x_2 - 0.961x_3 &> 0 \end{aligned} \quad (37)$$

From (37) we obtain:

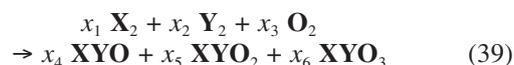
$$\begin{aligned} 4x_2/3 + 0.961x_3/3 < x_1 < 3x_2/2 + 0.474x_3/2, \\ x_2 > x_3/2 \end{aligned} \quad (38)$$

The inequalities (38) are necessary and sufficient conditions to hold the general reaction (36), *i.e.*, the reaction (36) holds if and only if (38) are satisfied.

*Example 1.* For instance, if we substitute  $\mathbf{X} = \text{Fe, Mn, Pb}$  in (1), we immediately obtain three sub-particular  $\aleph$  balanced reactions.

Next, we shall consider the following two  $\aleph$  reactions.

**Reaction 2.** Let us balance this general  $\aleph$  reaction with two arbitrary elements:



The above chemical reaction (39) reduces to the following system of linear equations:

$$\begin{aligned} 2x_1 &= x_4 + x_5 + x_6, \\ 2x_2 &= x_4 + x_5 + x_6, \\ 2x_3 &= x_4 + 2x_5 + 3x_6 \end{aligned} \quad (40)$$

Since the system (40) has three linear equations and six unknowns, we can solve it in  $6!/[(3!)(6-3)!] = 20$  ways. Actually, we must determine all the possible general solutions of the system (40). They are the following triads:  $(x_1, x_2, x_3)$ ,  $(x_1, x_2, x_4)$ ,  $(x_1, x_2, x_5)$ ,  $(x_1, x_2, x_6)$ ,  $(x_1, x_3, x_4)$ ,  $(x_1, x_3, x_5)$ ,  $(x_1, x_3, x_6)$ ,  $(x_1, x_4, x_5)$ ,  $(x_1, x_4, x_6)$ ,  $(x_1, x_5, x_6)$ ,  $(x_2, x_3, x_4)$ ,  $(x_2, x_3, x_5)$ ,  $(x_2, x_3, x_6)$ ,  $(x_2, x_4, x_5)$ ,  $(x_2, x_4, x_6)$ ,  $(x_2, x_5, x_6)$ ,  $(x_3, x_4, x_5)$ ,  $(x_3, x_4, x_6)$ ,  $(x_3, x_5, x_6)$  and  $(x_4, x_5, x_6)$ .

Since the size of our article is limited, we shall determine only one general solution of the system (40). It is the solution  $(x_1, x_2, x_3)$ .

1° Let  $x_4$ ,  $x_5$  and  $x_6$  be arbitrary real numbers, then the general solution of the system (40) is:

$$\begin{aligned} x_1 = x_2 &= (x_4 + x_5 + x_6)/2, \\ x_3 &= (x_4 + 2x_5 + 3x_6)/2 \end{aligned} \quad (41)$$

After the substitution of (41) into (39), the balanced reaction obtains this general form:

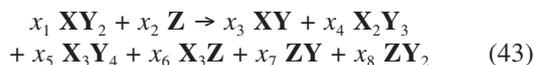
$$\begin{aligned} [(x_4 + x_5 + x_6)/2] \mathbf{X}_2 &+ [(x_4 + x_5 + x_6)/2] \mathbf{Y}_2 \\ &+ [(x_4 + 2x_5 + 3x_6)/2] \mathbf{O}_2 \\ \rightarrow x_4 \mathbf{XYO} &+ x_5 \mathbf{XYO}_2 + x_6 \mathbf{XYO}_3 \end{aligned} \quad (42)$$

$$\forall x_4, x_5, x_6 \in \aleph.$$

For reaction (39) to be fully balanced, the remaining 19 triads must be determined.

*Example 2.* For  $\mathbf{X} = \text{H}$  and  $\mathbf{Y} = \text{Cl}$ , we obtain a sub-particular reaction.

**Reaction 3.** Now we shall balance this general  $\aleph$  reaction with three arbitrary elements:



The above chemical reaction (43) reduces to the following system of linear equations:

$$\begin{aligned} x_1 &= x_3 + 2x_4 + 3x_5 + 3x_6, \\ 2x_1 &= x_3 + 3x_4 + 4x_5 + x_7 + 2x_8, \\ x_2 &= x_6 + x_7 + x_8 \end{aligned} \quad (44)$$

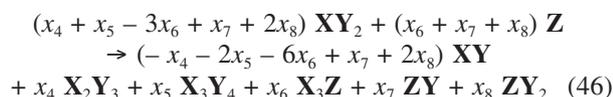
Since the system (44) has three linear equations and eight unknowns, we can solve it in  $8!/([3!(8-3)!]) = 56$  ways. Actually, we must determine all the possible general solutions of the system (44). They are the following triads:  $(x_1, x_2, x_3)$ ,  $(x_1, x_2, x_4)$ ,  $(x_1, x_2, x_5)$ ,  $(x_1, x_2, x_6)$ ,  $(x_1, x_2, x_7)$ ,  $(x_1, x_2, x_8)$ ,  $(x_1, x_3, x_4)$ ,  $(x_1, x_3, x_5)$ ,  $(x_1, x_3, x_6)$ ,  $(x_1, x_3, x_7)$ ,  $(x_1, x_3, x_8)$ ,  $(x_1, x_4, x_5)$ ,  $(x_1, x_4, x_6)$ ,  $(x_1, x_4, x_7)$ ,  $(x_1, x_4, x_8)$ ,  $(x_1, x_5, x_6)$ ,  $(x_1, x_5, x_7)$ ,  $(x_1, x_5, x_8)$ ,  $(x_1, x_6, x_7)$ ,  $(x_1, x_6, x_8)$ ,  $(x_1, x_7, x_8)$ ,  $(x_2, x_3, x_4)$ ,  $(x_2, x_3, x_5)$ ,  $(x_2, x_3, x_6)$ ,  $(x_2, x_3, x_7)$ ,  $(x_2, x_3, x_8)$ ,  $(x_2, x_4, x_5)$ ,  $(x_2, x_4, x_6)$ ,  $(x_2, x_4, x_7)$ ,  $(x_2, x_4, x_8)$ ,  $(x_2, x_5, x_6)$ ,  $(x_2, x_5, x_7)$ ,  $(x_2, x_5, x_8)$ ,  $(x_2, x_6, x_7)$ ,  $(x_2, x_6, x_8)$ ,  $(x_2, x_7, x_8)$ ,  $(x_3, x_4, x_5)$ ,  $(x_3, x_4, x_6)$ ,  $(x_3, x_4, x_7)$ ,  $(x_3, x_4, x_8)$ ,  $(x_3, x_5, x_6)$ ,  $(x_3, x_5, x_7)$ ,  $(x_3, x_5, x_8)$ ,  $(x_3, x_6, x_7)$ ,  $(x_3, x_6, x_8)$ ,  $(x_3, x_7, x_8)$ ,  $(x_4, x_5, x_6)$ ,  $(x_4, x_5, x_7)$ ,  $(x_4, x_5, x_8)$ ,  $(x_4, x_6, x_7)$ ,  $(x_4, x_6, x_8)$ ,  $(x_4, x_7, x_8)$ ,  $(x_5, x_6, x_7)$ ,  $(x_5, x_6, x_8)$ ,  $(x_5, x_7, x_8)$  and  $(x_6, x_7, x_8)$ .

As we mentioned previously, the size of the article is limited, and so we shall determine only one general solution for the system (44). It is the solution  $(x_1, x_2, x_3)$ .

1° Let us assume  $x_4, x_5, x_6, x_7$  and  $x_8$  are arbitrary real numbers, then the general solution of the system (44) is:

$$\begin{aligned} x_1 &= x_4 + x_5 - 3x_6 + x_7 + 2x_8, \\ x_2 &= x_6 + x_7 + x_8, \\ x_3 &= -x_4 - 2x_5 - 6x_6 + x_7 + 2x_8 \end{aligned} \quad (45)$$

After the substitution of (45) into (43), the balanced reaction obtains this general form:



Since the generators  $x_1, x_3 > 0$ , these inequalities should immediately follow:

$$\begin{aligned} x_4 + x_5 - 3x_6 + x_7 + 2x_8 &> 0, \\ -x_4 - 2x_5 - 6x_6 + x_7 + 2x_8 &> 0 \end{aligned} \quad (47)$$

From (47) we obtain:

$$x_7 + 2x_8 > x_4 + 2x_5 + 6x_6 \quad (48)$$

The above inequality (48) is a necessary and sufficient condition to hold the general reaction (46), *i.e.*, the reaction (46) holds if and only if (48) is satisfied.

For reaction (43) to be fully balanced the remaining 55 triads must be determined.

*Example 3.* For  $\mathbf{X} = \text{Mn}$   $\vee$   $\text{Fe}$ ,  $\mathbf{Y} = \text{O}$ , and  $\mathbf{Z} = \text{C}$  we obtain a sub-particular reaction.

### 3 DISCUSSION

The  $\aleph$  chemical reactions are a special kind of reactions that have non-unique coefficients. In chemistry, until now, they were balanced like a reaction with an infinite number of coefficients, which is incorrect. Every  $\aleph$  reaction has  $n!/([k!(n-k)!])$  general reactions, where  $n$  is the number of reaction molecules and  $k$  is the number of reaction elements. Each of these general reactions has an infinite number of sets of coefficients. In other words, every  $\aleph$  reaction reduces to  $n!/([k!(n-k)!])$  general reactions with an infinite number of particular sub-reactions for each of them.

In this article we determined all the general reactions of the reaction (1), which are given by the expressions (4), (6), (10), (13), (17), (20), (24), (28), (32) and (36). Also, for all of them we determined the necessary and sufficient conditions for which they hold. In three examples we showed that this approach to the balancing of  $\aleph$  reactions works successfully. We would also like to mention that the examples 1, 2 and 3 are derived sub-particular reactions, which are not fully balanced. The readers can derive the other general solutions very easily, because they are similar to those of reaction (1), which we derived using the technique of generalized algebra.

### 4 CONCLUSION

In this article three  $\aleph$  general chemical reactions are balanced. All the chemical reactions looked similar to *elementary* molecular reactions, but they were very hard to balance. Using this method of generalized algebra, the author proved again that balancing chemical reactions does not have anything to do with chemistry – it is a purely mathematical issue.

The strengths of the method of generalized algebra are:

1. This method provides an alternative approach for balancing  $\aleph$  chemical reactions. This method showed that matrix methods can be substituted by the method of generalized algebra.
2. Since this method of generalized algebra is well formalized, it belongs to the class of consistent methods for balancing chemical reactions.
3. This method of generalized algebra showed that for any  $\aleph$  chemical reaction a topology of its solutions can be introduced.
4. In fact, the offered method of generalized algebra simplifies the mathematical operations provided by the previous well-known matrix methods and is very suitable for daily practice. The method of generalized algebra has this advantage, because it fits for all  $\aleph$

chemical reactions, which previously were only balanced by the methods of generalized matrix inverses.

5. For a determination of general reactions any method for the solution of a system of linear equations can be used.
6. Using this method the general forms of the balanced chemical reactions are determined much faster than by other matrix methods.
7. From the general balanced reactions the other particular and sub-particular reactions can be determined.
8. Using the method of generalized algebra the dimension of the solution space can be determined.
9. Using this method the basis of the solution space can be determined.
10. Necessary and sufficient conditions for which some reaction holds can be determined by this method as well. These conditions determine the possibility of the reaction interval.
11. This method gives an opportunity to be extended with other numerical calculations necessary for  $\aleph$  reactions.
12. The method of generalized algebra represents a good basis for building a software package.  
The weak sides of the method are:
  1. Using this method the minimal reaction coefficients cannot be determined.
  2. This method cannot recognize when a chemical reaction reduces to one generator reaction.
  3. It cannot predict quantitative relations among the reaction coefficients.
  4. This method cannot arrange the molecules' disposition.
  5. The method of generalized algebra cannot predict reaction stability.

This method opens doors in chemistry and mathematics for new research on  $\aleph$  chemical reactions, which unfor-

tunately today cannot be balanced using a computer, because there is not such a method. The method of generalized algebra creates a large challenge for researchers to extend and adapt its usage for computer application. This is not an easy and simple job, but it deserves to be realized as soon as possible.

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